A Mass Formula from Light to Hypernuclei

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Simultaneous description of ordinary and hypernuclei masses by a single mass formula has been a great challenge in nuclear physics. Hyperon-separation energies of about forty Lambda(Λ), three Lambda-Lambda($\Lambda\Lambda$), one Sigma(Σ) and seven Cascade(Ξ) hypernuclei have been experimentally found. Many of these nuclei are of light masses. We prescribe a new mass formula, called BWMH, which describes the normal and hypernuclei on the same footing. It is based on the modified-Bethe-Weizsäcker mass formula (BWM). BWM is basically an extension of the Bethe-Weizsäcker mass formula (BW) for light nuclei. The parameters of BWM were optimized by fitting about 3000 normal nuclei available recently. The original Bethe-Weizsäcker mass formula (BW) was designed for medium and heavy mass nuclei and it fails for light nuclei. Two earlier works on hypernuclei based on this BW show some limitations. The BWMH gives improved agreement with the experimental data for the line of stability, one-neutron separation energy versus neutron number spectra of normal nuclei, and the hyperon-separation energies from hypernuclei. The drip lines are modified for addition of a Λ hyperon in a normal nucleus.

Keywords : Hypernuclei, Separation Energy, Dripline nuclei, Mass formula, Hyperon-nucleon interaction.

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1. Introduction

In the last two decades scientific activities have been focussed on hypernuclear physics which is at the boundary between nuclear and particle physics. To obtain a comprehensive view of the basic properties and fundamental interactions of the hadronic system many sophisticated experiments

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have been done in these interdisciplinary fields. Separation energies have been determined for the ground states of about 40 Λ hypernuclei, three double-Λ hypernuclei [1], [2], several Ξ^-(S=-2) hypernuclei [1] and only one bound Σ hypernucleus. Several studies suggest that due to strongly repulsive Σ-nucleus potential Σ's are unbound in nuclei, except for the very special case of nuclei with mass number A=4 [3]. No bound state data exists on the Θ^+ (S=+1, mass $\sim$1530 MeV, width $<$15 MeV) hypernucleus with exotic pentaquark existence of which was predicted in 1997 [4] and announced in 2003 at Spring-8, Japan [5] along with several claims of nonexistences [6]. Calculations in a relativistic mean-field formalism (RMF) suggest that as there is an attractive Θ^+-nucleus interaction, the Θ^+ particle can be bound in nuclei and, the Θ^+ hypernuclei would be bound more strongly than Λ hypernuclei [7]. Searches for more experimental data on bound Θ^+ hypernuclei are on for a large number of hypernuclei, including Λ, $\Lambda\Lambda$, Ξ and Σ hyperons. A single mass formula [8] for both the strange and nonstrange nuclei has been formulated to predict the binding energies of all of them on the same footing. Its predictions compare well with the available experimental data. It is not applicable for repulsive potential.

2. Generalised mass formula for non-strange and strange nuclei

Generalization of mass formula was pursued starting from the modified-Bethe-Weizsäcker mass formula (BWM) preserving the normal nuclear matter properties[9]. The BWM is basically the Bethe-Weizsäcker mass formula extended for light nuclei [10] which can explain the gross properties of binding energy versus nucleon number curves of all non- strange normal nuclei from Z=3 to Z=83. A systematic search of experimental data of hyperon separation energy (S_Y) for Λ, $\Lambda\Lambda$, Σ^0 and Ξ^- hypernuclei leads to a generalised mass formula (BWMH) for hyper and non-strange nuclei. The hypernucleus is considered as a core of normal nucleus plus the hyperon(s). Strangeness and hyperon-mass dependent terms are explicitly included in BWMH breaking the $SU_F(3)$ symmetry and the binding energy is given as

$$\begin{aligned} B(A,Z) = \quad & 15.777A - 18.34A^{2/3} - 0.71\frac{Z(Z-1)}{A^{1/3}} - \frac{23.21(N-Z_c)^2}{[(1+e^{-A/17})A]} \\ & +(1-e^{-A/30})\delta + n_Y[0.0335(m_Y) - 26.7 - 48.7 \mid S \mid A^{-2/3}], \end{aligned} \tag{1}$$

where $\delta = 12A^{-1/2}$ for N, Z_c even, $= -12A^{-1/2}$ for N, Z_c odd, $= 0$ otherwise, $n_Y =$ number of hyperons in a nucleus, $m_Y =$ mass of the hyperon in MeV, $S =$ strangeness of the hyperon and mass number $A = N + Z_c + n_Y$ is

equal to the total number of baryons. N and Z_c are the number of neutrons and protons respectively while the Z in eqn.(1) is given by $Z = Z_c + n_Y q$ where q is the charge number (with proper sign) of hyperon(s) constituting the hypernucleus. For non-strange (S=0) normal nuclei, $Z_c = Z$ as n_Y =0. The choice of δ value depends on the number of neutrons and protons in both normal and hypernuclei. For example, in case of ${}^9_\Lambda Li$, $\delta = -12A^{-1/2}$ as the (N, Z_c) combination is odd-odd, whereas, for non-strange normal ${}^9Li$ nucleus $\delta = 0$ for A=9(odd).

The hyperon separation energy S_Y defined as

$$S_Y = B(A, Z)_{hyper} - B(A - n_Y, Z_c)_{core}, \tag{2}$$

is the difference between the binding energy of a hypernucleus and the binding energy of its non-strange core nucleus. On the other hand, the separation energies of single neutron (S_n) and proton (S_p) from the hypernuclei containing single Λ inside the nucleus are defined as

$$S_n = B(A, Z)_{hyper} - B(A-1, Z)_{hyper},\ S_p = B(A, Z)_{hyper} - B(A-1, Z-1)_{hyper}. \tag{3}$$

It is interesting to note that the values of S_Y using eqn.(2) are in reasonable agreement with the available experimental data of all known bound hypernuclei. Fig.[1] shows plots of S_Y versus A for Λ and $\Lambda\Lambda$, Ξ^- and Θ^+ hypernuclei. BWMH predictions of the first three are in good agreement with the available experimental data [1, 2] and the same for the last one are in close agreement with the quark mean field (QMF) calculations [11]. Available experimental binding energy values for Σ^+ binding energy in ${}^4_\Sigma He$ are $4.4 \pm 0.3 \pm 1MeV$ [12], $2.8 \pm 0.7MeV$ [13], $4 \pm 1MeV$ [14]. The BWMH predicts binding energy of Σ^0 ($m_Y = 1192.55MeV$) and Σ^+ ($m_Y = 1189.37MeV$) in ${}^4_{\Sigma^0}He$ (=Σ^0 + ${}^3_2He$) as $2.69MeV$ and ${}^4_{\Sigma^+}He$ (=Σ^+ + ${}^3_1H$) as $1.6MeV$ respectively. Search for bound Σ hypernuclei has led to the conclusion that a Σ-nucleus potential is strongly repulsive [3] excepting ${}^4_\Sigma He$ and without changing any parameter BWMH reproduces the binding energy of the ${}^4_\Sigma He$ hypernucleus. For $\Sigma^0 +^2 H$, $\Sigma^+ +^2 H$, and $\Sigma^- +^2 H$ hypernuclei the separation energies predicted by BWMH are -3.53 MeV, -4.62 MeV and -3.37 MeV respectively indicating that these light Sigma hypernuclei would be unbound, even if the potential is attractive. So far no bound state of these hypernuclei could be found in the experiment. These observations suggests that further data on Σ-hypernuclei are necessary to determine more conclusively whether the Σ feels attraction or repulsion.

The effect of addition of a single Λ in a non-strage normal nucleus can be seen through the one-neutron and one-proton separation energies tabulated in Table 1. Since hypernuclei are more bound than normal nuclei as a result of increase of nuclear potential depth, drip lines for single Λ hypernuclei

spread out on the either sides. This observation may have important consequences in the astrophysical objects where bound hypernuclei may exist and form strange stars.

Earlier, Dover and Gal [15] prescribed two separate mass formulae for Λ and Ξ hypernuclei by introducing several volume and symmetry terms in Bethe-Weizsäcker mass formula (BW), but it did not reproduce experimental data. Mass formula proposed by Levai et al. [16] gave reasonable description of the experimental data on Λ and $\Lambda\Lambda$ hyperon(s) separation energies, but the binding energy per nucleon diverges as mass number A goes to infinity. Since none of them contain explicit hyperon mass in their formulae, they can not be used for binding energy calculation of other hypernuclei. BWMH is not plagued with such divergences and nuclear saturation properties are well preserved for large A.

3. Summary and Conclusion

In summary, a simple one line mass formula (BWMH) applicable to normal as well as strange hypernuclei is developed by introducing hyperon mass and strangeness dependent SU(6) symmetry breaking terms in BWM. Since it is not applicable for repulsive potential it does not predict negative sigma separation energy for heavier Σ-nuclei. It predicts that Θ hypernuclei would be more strongly bound than Λ hypernuclei which is in good agreement with the quark mean field calculation [11]. Calculations of S_p and S_n of normal nuclei and nuclei with single Λ hyperon inside the nucleus indicate that due to stronger Λ-nucleon interaction, the mean field potential gets modified by addition of single Λ hyperon to the core. Introduction of Λ inside a nucleus thus alters the usual neutron and proton driplines and gives birth of new nuclei of astrophysical interest beyond normal neutron and proton driplines.

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Table 1. One-nucleon separation energies on driplines for each element with the lowest and highest number of bound neutrons in normal [17] and Λ-hypernuclei.

At. No. Z	Normal p-drip N, S_p	Normal n-drip N, S_n	Hyper p-drip N, S_p	Hyper n-drip N, S_n	At. No. Z	Normal p-drip N, S_p	Normal n-drip N, S_n	Hyper p-drip N, S_p	Hyper n-drip N, S_n
3	2, 3.36	8, .58	1, 1.22	8, 1.87	4	2, 1.11	10, .90	2, 3.79	10, 1.84
5	3, .74	12, .99	3, 2.39	12, 1.73	6	3, .17	14, 1.01	3, 1.80	14, 1.63
7	5, 1.98	16, .97	4, .24	16, 1.50	8	5, 1.98	18, .94	4, .44	18, 1.40
9	6, .09	20, .89	6, .81	22, .01	10	6, .64	22, .87	6, 1.39	24, .04
11	8, .54	24, .84	8, 1.05	26, .08	12	8, 1.37	26, .84	7, .05	28, .13
13	10, .71	28, .84	10, 1.10	30, .18	14	9, .07	30, .86	9, .56	32, .24
15	12, .74	34, .04	12, 1.06	34, .31	16	11, .46	36, .12	11, .86	36, .37
17	14, .68	38, .21	14, .95	38, .44	18	13, .69	40, .29	13, 1.02	40, .51
19	16, .57	42, .37	16, .80	42, .58	20	15, .81	44, .45	15, 1.08	46, .07
21	18, .42	46, .53	18, .62	48, .17	22	17, .84	50, .07	17, 1.08	50, .25
23	20, .24	52, .17	20, .42	52, .34	24	19, .81	54, .25	19, 1.02	54, .41
25	22, .04	56, .34	22, .20	58, .05	26	21, .73	58, .41	21, .92	60, .13
27	25, .76	62, .08	25, .89	62, .22	28	23, .61	64, .16	23, .78	64, .29
29	27, .48	66, .24	27, .60	68, .00	30	25, .47	68, .31	25, .62	70, .08
31	29, .19	72, .04	29, .30	72, .16	32	27, .30	74, .11	27, .44	74, .22
33	32, .66	76, .18	31, .00	76, .30	34	29, .11	78, .25	29, .23	80, .05
35	34, .33	82, .02	34, .42	82, .12	36	32, .62	84, .08	31, .02	84, .19
37	36, .00	86, .15	36, .08	86, .25	38	34, .36	88, .21	34, .46	90, .04
39	39, .33	92, .01	39, .40	92, .10	40	36, .10	94, .07	36, .19	94, .16
41	42, .58	96, .13	41, .04	96, .22	42	39, .45	98, .19	39, .53	100, .03
43	44, .20	102, .01	44, .26	102, .09	44	41, .14	104, .07	41, .22	104, .15
45	47, .40	106, .13	47, .45	106, .20	46	44, .39	108, .18	44, .46	110, .03
47	49, .01	112, .02	49, .06	112, .09	48	46, .06	114, .07	46, .12	114, .14
49	52, .14	116, .12	52, .18	116, .19	50	49, .25	118, .17	49, .30	120, .04
51	55, .24	122, .03	55, .28	122, .09	52	52, .38	124, .07	52, .43	124, .14
53	58, .30	126, .13	58, .33	128, .00	54	54, .02	128, .17	54, .07	130, .05
55	61, .34	132, .04	61, .37	132, .10	56	57, .11	134, .08	57, .16	134, .14
57	64, .35	136, .13	64, .38	138, .02	58	60, .17	140, .01	60, .21	140, .06
59	67, .35	142, .05	67, .37	142, .11	60	63, .21	144, .10	63, .25	144, .15
61	70, .32	146, .14	70, .34	148, .04	62	66, .22	150, .03	66, .25	150, .08
63	73, .28	152, .07	73, .30	152, .13	64	69, .22	154, .11	69, .25	156, .01
65	76, .22	158, .01	76, .24	158, .06	66	72, .19	160, .05	72, .22	160, .10
67	79, .15	162, .09	79, .17	162, .14	68	75, .15	164, .13	75, .18	166, .04
69	82, .07	168, .03	82, .08	168, .08	70	78, .10	170, .07	78, .12	170, .12
71	86, .31	172, .11	86, .32	174, .02	72	81, .03	176, .02	81, .05	176, .06
73	89, .20	178, .06	89, .21	178, .10	74	85, .27	180, .10	85, .29	182, .01
75	92, .08	184, .01	92, .08	184, .05	76	88, .17	186, .04	88, .18	186, .09
77	96, .25	188, .08	96, .25	190, .00	78	91, .07	190, .12	91, .08	192, .04
79	99, .11	194, .04	99, .12	194, .08	80	95, .24	196, .07	95, .25	196, .11
81	103, .25	198, .11	103, .25	200, .03	82	98, .11	202, .03	98, .12	202, .07
83	106, .08	204, .07	106, .09	204, .10					

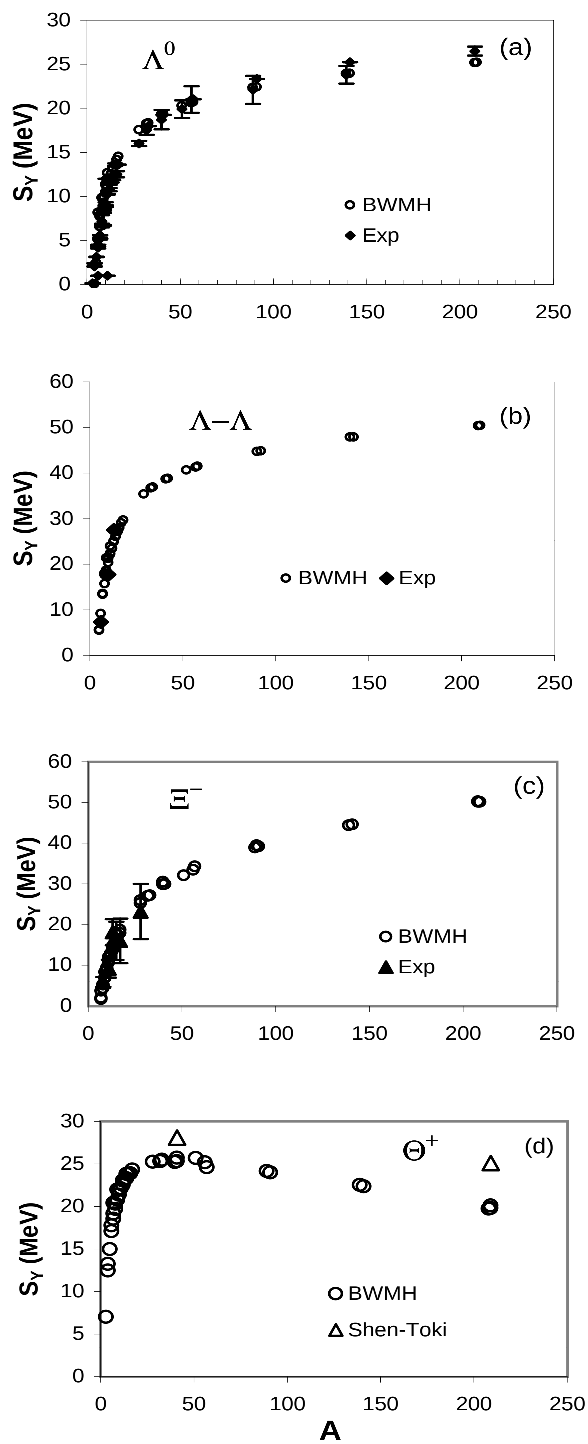

Fig. 1. Plots of hyperon-separation energies of (a) Λ,(b) $\Lambda\Lambda$, (c) Ξ^- and (d) Θ^+ hypernuclei with mass number(A). BWMH predictions are compared with experimental data for Λ, $\Lambda\Lambda$, Ξ^- and QMF predictions [11] for the three Θ^+ hypernuclei.